Ratios, Proportions, and Percents Review

Ratios are a pair of numbers that compare different types of units.
3 ways to write them:

$$\frac{1}{2} \qquad\qquad 1 \text{ to } 2 \qquad\qquad 1:2$$

Equivalent Ratios – ratios that make the same comparison
Ex. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent ratios

Proportion – an equation showing how 2 ratios are equal
Ex. $\frac{2}{3} \qquad \frac{8}{12}$

Solving Proportions – when there is a missing number in a proportion, we can use **cross multiplication** to find the missing value. (Yes, you can use the fish method, too!) ;)
Ex.

$$\frac{a}{b} \times \frac{c}{d} \qquad\qquad \frac{2}{4} \times \frac{x}{10} \qquad\qquad \frac{2}{4} = \frac{x}{10}$$

$$ad = bc \qquad\qquad 4x = 20 \qquad\qquad \frac{20}{4} = x$$

$$\frac{4x}{4} = \frac{20}{4} \qquad\qquad x = 5$$

$$x = 5$$

Unit Rate - when a ratio or rate is simplified so that the denominator is 1 unit.
Ex.

$$\frac{\#\,miles}{1\ gallon} \qquad\qquad \frac{\#\,dollars}{1\ hour}$$

How to find the unit rate:
1) Set up your ratio as a fraction, then reduce so the denominator is 1.

$$\frac{45\,miles}{3\,hours} \frac{\div 3}{\div 3} = \frac{15\,miles}{1\,hour}$$

2) Divide the numerator by the denominator.

$$\frac{45\,miles}{3\,hours} \qquad \frac{45}{3} = 15 \qquad \frac{15\,miles}{1\,hour}$$

A unit price is a rate that gives the cost of an item per 1 unit of that item.
If you have money involved in a unit rate, the amount of money is always the numerator!

$$\frac{\$3.96}{18\,oz.} = \frac{\$0.22}{1\,oz.}$$

Proportions with Tables:

Time (sec.)	1	2	3	4	5
Distance (in.)	5	10	15	20	25
Ratio:	$\frac{5}{1}$	$\frac{10}{2}=\frac{5}{1}$	$\frac{15}{3}=\frac{5}{1}$	$\frac{20}{4}=\frac{5}{1}$	$\frac{25}{5}=\frac{5}{1}$

For each column, find the ratio of the distance to the time in it's simplest form. If it is a proportion, that ratio will be the same for each column. That ratio is called the **constant of proportionality (k).** So… you can write an equation from the table as **y = kx**, where k is the ratio that you just found for each column in the table. The constant of proportionality is also the unit rate!

$$unit\ rate = \frac{5}{1} \qquad k = 5 \qquad y = 5x$$

Ex. 2 pounds of cashews cost $5, 3 pounds of cashews cost $7.50 and 8 pounds of cashews cost $20. Show that the relationship between the pounds of cashews and the cost is proportional and write an equation for the relationship.

Number of pounds	2	3	8
Cost ($)	$ 5	$ 7.50	$ 20

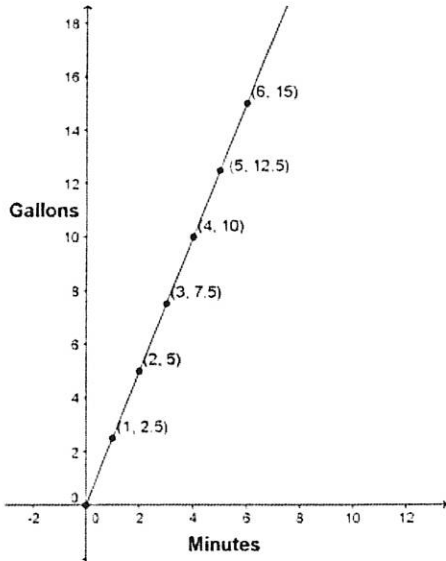unit rate → (constant of proportion.) $\frac{5}{2} = \frac{2.5}{1}$ $\frac{7.50}{3} = \frac{2.5}{1}$ $\frac{20}{8} = \frac{2.5}{1}$

$y = 2.5x$

Proportions and Graphs:

The graph is showing showerhead water use:

Time (min)	1	2	3	4	5
Water Used (gal)	2.5	5	7.5	10	12.5

ratio → $\frac{2.5}{1}$ $\frac{5}{2} = \frac{2.5}{1}$ $\frac{7.5}{3} = \frac{2.5}{1}$ $\frac{10}{4} = \frac{2.5}{1}$ $\frac{12.5}{5} = \frac{2.5}{1}$

Remember in a graph, the "y" depends on the "x" ("dry mix"). So with this table, does the amount of water used depend on how much time you shower, or does how much time you shower depend on the water used? Amount of water depends on how long you are showering, so... the amount of water is on the y-axis, while the time is on the x-axis.

**A graph shows a proportion if it is a straight line, and goes through the origin (0,0)!**

To write an equation from a graph, find the constant of proportionality (k). (Hint, it's $\frac{y}{x}$ in simplest form, or the unit rate!) Then, substitute your answer into the equation y = kx. For this graph, k = 2.5 , so your equation is y = 2.5x

To draw a graph from a table, figure out which value depends on the other one to determine your x and y axis. Plot the points on the graph from the table. Calculate additional points if needed. Connect your points.

Amount earned	$10	$20	$50	$100
Hours worked	1	2	5	10

The amount you earn depends on how much you work, so amount earned is on the y-axis, while hours worked is on the x-axis.
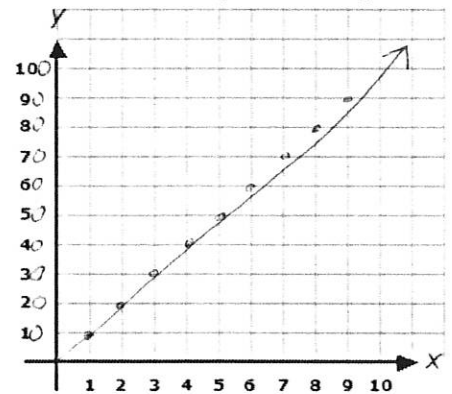
Fractions, Decimals, and Percents:

To convert decimals to percents:
1) Multiply the decimal by 100 (move the decimal 2 places to the right)

.55 = 55%

To convert percents to decimals:
1) Divide the percent by 100 (move the decimal 2 places to the left)

62% = .62

To convert fractions to percents:
1) Write as a proportion and solve. $\frac{9}{20}$

$\frac{9}{20} = \frac{n}{100}$

$\frac{900}{20} = \frac{20n}{20}$ $n = 45\%$

To convert percents to fractions:
1) Write the percent as a fraction and simplify.   125%

$\frac{125}{100} \overset{\div 25}{\underset{\div 25}{=}} \frac{5}{4} = 1\frac{1}{4}$

Percents:

Percent Proportion – compares a part of a quantity to the whole quantity, using a percent.

1) Write two ratios

$$\frac{\text{part (a)}}{\text{whole (b)}} = \frac{\text{percent}}{100}$$

$$\frac{a}{120} = \frac{40\%}{100}$$

2) Solve the proportion by cross multiplying
   part x 100 = whole x percent

$$\frac{a}{120} \underset{100}{\overset{40}{\times}}$$

$$100a = 4800$$
$$a = 48$$

Percent Equation – State the problem as a sentence where "is" becomes the = sign and "of" means to multiply.

   part is percent of base
   part = % x base

What # is 12% of 350

$$x = .12 \times 350$$
$$x = 42$$

13 is 26% of what #

$$13 = .26 \times A$$
$$\frac{13}{-26} = \frac{.26 \times A}{.26}$$
$$50 = A$$

Percent Change – the ratio of the amount of increase or decrease to the original amount

$$\frac{(\text{new} - \text{old})}{\text{old}} \times 100 = \% \text{ change}$$

If the answer is positive, it is an increase.
If the answer is negative, it is a decrease.

Ex. A pair of shoes were $60, but are currently $50. What is the percent change? Is it an increase or decrease?
How do you know?

$$\frac{50-60}{60} = -16.7\%$$

16.7% decrease, b/c negative #

Ex. A bakery raised the price of its cookies from $6 a dozen to $10 a dozen. What is the percent change? Is it an increase or decrease? How do you know?

$$\frac{10-6}{10} = 40$$

40% increase b/c positive #

Percent Error – compares a guess or estimate to an exact value

$$\frac{|\text{approximate value} - \text{exact value}|}{\text{exact value}} \times 100$$

Ex. I estimated 1000 people were in attendance at the concert, but there were actually 1550.

$$\frac{|\text{approximate value} - \text{exact value}|}{\text{exact value}} \times 100$$

$$\frac{|1000 - 1550|}{1550} = \frac{550}{1550} = 35.5\% \text{ error}$$

Markups / Discounts

If it's asking for **percent** markup or discount, solve like percent change:

$$\frac{(\text{new} - \text{old})}{\text{old}} \times 100$$

% markup if $10 shirt sells for $25?

$$\frac{25-10}{10} = 1.5 \times 100 = \boxed{150\% \text{ markup}}$$

If it's asking for **amount** of markup or discount, simply multiply the percent by the original amount.

IF $10 shirt is marked up 150% how much is shirt marked up?

$$1.50 \times \$10$$
$$= \boxed{\$15}$$

If it's asking for the **new price**, first find the amount of markup or discount, then add to (for markups) or subtract from (for discounts) the original price.

IF $10 shirt is marked up 150% how much is the new price?

$$1.5 \times \$10$$
$$= \$15$$

$$\begin{array}{r} 10 \\ + 15 \\ \hline \boxed{\$25} \end{array}$$

Sales Tax, Tip, Commissions, Fees

Calculate the same way you do to find the amount of markups.

1) Convert the percent to a decimal.
2) Multiply the decimal by the original amount to find the tax, tip, etc.
3) If the question is asking for the total price or total amount, add your amount of tax, tip, etc. to the original amount.

Ex. Shoes are $125
Sales tax = 7%

$.07 \times 125$
$= \$8.75$ in sales tax

$\begin{array}{r} \$125 \\ 8.75 \\ \hline \$133.75 \end{array}$ total amount

Interest

$I = prt$ Interest = principal x rate x time

Principal - amount of money borrowed or invested
Rate – amount charged to use the money (in percent)
Time – how long the money is borrowed or invested (in years)

Ex. How much interest for $8000 invested at 6% for 1 year?

$I = prt$

$I = 8000 \times .06 \times 1$

$I = \$480$

1) Change the percent to a decimal.
2) Substitute in the equation.
3) If time is in months, set up as a fraction with a denominator of 12.

Ex. 2. How much interest on $1200 charged at 21% rate for 1 month?

$I = prt$

$I = 1200 \times .21 \times (1/12)$

$I = \$21$